MTH 1420, SPRING 2012
DR. GRAHAM-SQUIRE

SECTION 5.6: INTEGRATION BY PARTS

HW: # 6, 12, 13, 16, 26, 39 (Hint for number 12- You will need to start out similar to Example 2 on page 384)

Practice: # 5, 9, 15, 23, 29, 33, 40

1. Introduction

The tricks we use to evaluate integrals all come from the rules we used for derivatives. As we saw in the last chapter, doing u substitution is just doing the chain rule backwards. Similarly, we have a rule for doing the *product rule* backwards, and it is called integration by parts.

2. Integration by Parts - Theory

Let's figure out the formula for integration by parts by taking the product rule and integrating it:

Theorem 2.1. *(Integration by Parts) The formula for doing integration by parts is*

$$\int u\,dv = uv - \int v\,du.$$

3. Integration by Parts - Method

It is not immediately clear from the formula above how one is supposed to do integration by parts, or why it would be useful. We will look at some examples to illustrate these things.

Example 1. Calculate $\displaystyle\int x\cos(5x)dx$.

Exercise 2. Calculate $\displaystyle\int_0^1 \frac{y}{e^{2y}}\,dy$. Evaluate it as if it was an indefinite integral (as we did in the example above), then plug in the limits of integration. Use $u = \frac{1}{e^{2y}}$ and $ydy = dv$.

Exercise 3. Calculate $\int_1^e x^2 \ln x \, dx$.

4. Integration by Parts - Tricky ones

There are times when one application of integration by parts does not solve the integral, and you have to do integration by parts again. Other times, it is not remotely clear what you should choose for u and what for dv- in these cases you have to make a best guess and see if it gives you an easier integral. If it does not, then you have to start again and try a different u and dv. See example 2 on page 384 in the textbook for a particularly strange use of integration by parts.

Example 4. Calculate $\int e^{-\theta} \cos(2\theta) \, d\theta$.

Exercise 5. Calculate $\int_0^1 \frac{r^3}{\sqrt{4+r^2}}\,dr$.